WISVORA

Available online at Wisvora

Journal of Global Governance and Sustainability

Journal homepage: https://wisvora.com/index.php/jggs/index

Municipal property pricing model: A sustainable viewpoint for urban and rural areas

Vidya Trisandini Azzizi [1], Jieh-Haur Chen [2]*, F.IET [2], Ting-Kwei Wang[3], and Tzuyang Yu, F.JSPS[4]

[1]Assistant Professor, Department of Urban and Regional Planning, Podomoro University.
[2]Distinguished Professor, Department of Civil Engineering; Director, Research Center of Smart Construction; and Associate Dean, College of Engineering, National Central University, Zhongli, Taoyuan 320317, Taiwan.
[3]Associate Professor, Department of Civil Engineering, National Kaohsiung University of Science and Technology, Kaohsiung 81164, Taiwan.
[4]Professor, Department of Civil and Environmental Engineering, University of Massachusetts, Lowell, MA 01854, USA.

ABSTRACT

This study aims to create a sustainable land pricing model based on Black-Scholes theory, integrating critical environmental and socio-economic factors to promote equitable development in both urban and rural areas. By conducting a comprehensive literature review spanning macroeconomics, environmental impact, and sustainable land management, a robust mathematical framework is developed using principles of financial engineering. The proposed model not only predicts housing prices with high accuracy (93.6% to 95%) but also delineates optimistic and pessimistic valuation boundaries while identifying regions with high developmental potential through trend indicators. Urban area price growth is forecasted between 2.1% and 21.5%, providing actionable insights for future-oriented planning. Beyond financial forecasting, this model incorporates sustainability metrics, such as land-use efficiency, ecological footprint, and green infrastructure potential, making it a strategic tool for climate-resilient urban planning. It supports real estate developers in evaluating environmentally responsible investment opportunities, while offering investors and policy-makers a decision-making instrument aligned with low-carbon and socially inclusive growth. The findings provide a foundation for region-specific, sustainability-oriented pricing strategies, fostering long-term economic viability, environmental stewardship, and informed governance in the real estate sector.

KEYWORDS

housing market, environmental features, municipal property, urban and rural.

---

1. INTRODUCTION

✧ This work was supported by the Taiwan Ministry of Science and Technology (MOST) / National Science and Technology Council (NSTC) under Grants MOST-108-2221-E-008 -002 -MY3, MOST-109-2622-E-008-018-CC2, MOST-110-2622-E-008-018-CC2, MOST-110-2221-E-008-052-MY3, NSTC-111-2622-E-008-017, and NSTC-111-2221-E-008-027-MY3

* *Corresponding author: Jieh-Haur Chen* (e-mail: jhchen@ncu.edu.tw)

Received September 30, 2025; Accepted October 23, 2025
Available online October 29, 2025

Continual advancements in transportation infrastructure aim to enhance regional and urban accessibility while alleviating local traffic congestion. This progress significantly impacts property owners and residents in these areas, directly influencing land prices [1]. Notably, urban housing investments, sales, and prices have experienced substantial growth, sometimes exceeding 20% annually in certain cities. Consequently, these changes have garnered considerable attention from governmental bodies, enterprises, specialists, and the media. Presently, predictions made by construction firms, banks, and government initiatives rely heavily on empirical methods to forecast future trends, posing investment risks in the absence of accurate trends. In rural settings, the construction of the arterial road led to a remarkable increase in land prices—up to 317% for agricultural land and 50% for drylands. The value of land intertwines with its use, and any factor impacting land productivity reflects in its market price [2]. Analyzing influential factors and their impact on housing prices is crucial as it furnishes essential guidance for navigating housing price trends. Typically, individuals turn to seasoned appraisal agents with years of experience to determine value, yet this method can be time-intensive and prompt frequent price adjustments lacking specific evaluation criteria. These adjustments, reliant on subjective experiences, introduce considerable deviations. This drawback amplifies for investors heavily committed to a region, posing immediate business risks stemming from uncertain housing price assessments. Developing a specialized land pricing tool for specific regions becomes imperative to minimize undue losses arising from uncertain evaluations.

The study aims to achieve two primary objectives: firstly, to construct a real estate pricing model within a specified region by incorporating Black and Scholes theory alongside environmental concepts; and secondly, to illustrate its usability across urban and rural areas. This model could potentially serve as a valuable instrument for investors, facilitating the analysis of factors and forecasting future price trends. Given its integration of environmental features, the outcomes concentrate on delivering a mathematical function, regional price trends, and delineated price boundaries. Moreover, the model's ability to generate specific prices for individual real estate properties can be beneficial as a point of reference when applicable.

2. PROPERTY APPRAISAL APPROACHES AND MODELS

Theoretically, the value of land across various uses reflects the demand for the specific property type built upon it relative to the available land supply for that particular use. However, practical valuation is intricate due to multiple factors, such as market discontinuities (including those influenced by topography), investment and lending trends, transport accessibility, development potential, planning regulations, and other variables. These elements collectively form a complex and irregular landscape of property values. Consequently, land value diverges between urban and rural areas due to distinct factors. Urban regions predominantly feature trading and commercial land uses, while rural areas focus on agricultural purposes, emphasizing the importance of the land's physical attributes. Moreover, any alterations in land use within urban locales—stemming from government or private investments in infrastructure and facilities—tend to impact land prices, often resulting in an increase. Consequently, a trend emerges where land prices in a specific area typically rise when it is closer to the functional core of an urban area [3].

Individual property appraisal typically follows several prevalent approaches: cost basis, sale comparison, income basis, and the hedonic model. However, the first three methods heavily rely on the appraisers' experience and are susceptible to introducing biases into the appraisal process. In contrast, the hedonic price theory is founded on heterogeneous goods characterized by a range of attributes that contribute to their utility and overall value. This theory asserts that the transaction price in the real market is a composite of implicit prices or hedonic prices associated with these distinct characteristics. Essentially, each characteristic of a product corresponds to an implicit price, creating multiple implicit markets that collectively form the total market. While market transactions are observable, these implicit prices often remain concealed. In real estate pricing literature, two dominant methods prevail: the hedonic regression method and the repeat sales method [4]. Real estate, being a complex commodity, can exhibit varied selling prices influenced by factors like environmental quality, accessibility, among others. The Black-Scholes (1973) option pricing model, incorporating various derivatives and securities, has been extensively employed despite empirical studies highlighting deficiencies in the formula. These deficiencies stem from simplified assumptions, including geometric Brownian motion of stock return, constant variance, absence of taxes or continuous trading on underlying assets, which often don't hold in financial markets [5]. An extension of this model, known as the Extension of the Black–Scholes model (EBS), delves into the interdependency structure of multiple assets in financial markets, offering deeper financial and mathematical insights [6]. This extension provides conditions for the existence of an equivalent martingale measure, completeness, and techniques for pricing contingent claims on various assets.

Although research on the spatial heterogeneity of factors influencing land prices has been relatively limited, numerous studies indicate the complexity of these factors and the challenge in accurately pinpointing their impact on land prices. Crucially, land prices wield significant influence in guiding urban planning and resource allocation, especially in rapidly evolving cities of developing nations characterized by dynamic infrastructures and shifting populations. Detecting spatially implicit information about the relationship patterns between land prices and associated influencing factors becomes imperative. Geographically Weighted Regression (GWR), an emerging local spatial statistical method, has gained traction in recent years for its ability to assess how the relationships between dependent and explanatory variables vary across space. There's been a growing body of literature focused on residual analysis, stationary tests, arithmetic exploration, and other theoretical studies related to GWR due to its evident advantages, leading to widespread utilization across various fields. Examples include studies on geographic variations in land use change, deforestation, burglary, childhood drowning, recreation demand, urban space, male suicide, municipal water consumption, and population mapping. The hedonic pricing model, using market goods as proxies wherein non-market elements like accessibility are implicitly traded, describes product prices based on their hedonic features [7]. Scholars like Roback (1982) and Rosen (1979) emphasize the influence of comfort on labor and housing markets as people select cities based on living and working conditions. Urban housing prices embody various comfort properties—physical, economic, and social services environments, among others—affecting worker mobility and resulting in varied housing prices [8]. However, the hedonic technique demands stringent assumptions about the preferences of market participants. It faces challenges when attempting to aggregate preferences across diverse consumers, suggesting limitations when applied to analyzing rural property sales data. Consequently, the current hedonic model and its application may fall short in capturing transaction patterns [9].

According to the hedonic price theory, the relation between product price and product characteristics can be expressed as follows:

$$P = H(c_1, c_2, \ldots, c_n) \tag{1}$$

where P is product price, c is product characteristic, n stands for an integral, and H represents product characteristic function. We can obtain the corresponding implicit prices of characteristics by calculating the derivative of the function with understanding of each characteristic variable. The hedonic price equation can be formulated:

$$P_i = \frac{\partial P}{\partial c_i}, \ for\ i = 1, 2, \ldots, n \tag{2}$$

The hedonic price model operates on certain assumptions: heterogeneity among targeted objects, a uniform market, implicit market features, and market equilibrium. Compared to current pricing methods, this model offers a consumer-centric approach to house pricing, accommodating diverse housing requirements and resulting in more reasonable purchasing costs and improved investment recovery. In contrast, mass appraisal, evolving rapidly since the 1970s, amalgamates statistical, mathematical, and related techniques. Widely adopted in real estate tax assessments in Western nations, the mass appraisal system stands as a critical valuation method resting on three pillars: assessment theory methods, mathematical statistics, and computer application technologies. Presently, it holds the status of the most significant real estate valuation method.

3. ESTABLISHMENT OF PROPERTY PRICING MODEL

The Black and Scholes option-pricing model marked a significant breakthrough in derivative pricing several decades ago. Various studies have scrutinized its performance in pricing, with references including Black (1973), Emanuel (1982), Macbeth (1979), Morris and Limon (2010), Rubinstein (1985), and Singh et al. (2011). Extended and modified versions of this model have been effectively employed in valuing options, as demonstrated by studies such as Morris and Limon (2010) and Singh et al. (2011). While discussions on option thetas are prevalent in finance-related literature and derivative papers, thorough examinations of their properties remain scarce. Efficient numerical differentiation methods have been utilized to compute option thetas, particularly focusing on call options and breaking down a call option into a margin value and an insurance policy [10, 11]. Earlier research explored the option effect, underlying effect, and curve effect in an option theta [12]. Visual representations of the Black-Scholes model depict option thetas and time premiums, as observed in works by

Albeverio et al. [6] and Emery et al. [13].

The Black-Scholes model, renowned for option pricing, has found application beyond financial domains. Investigations into wind energy forecast error estimation revealed the model's efficacy in not just pricing options but also in gauging potential errors in wind energy predictions for future time frames. This methodology harnessed the concept of historical volatility from finance and adapted it to gauge wind energy's historic volatility, aiding in error estimation for wind energy forecasts [14]. Additionally, scholars explored perpetual American option pricing by employing a self-financing delta-hedging strategy. This approach aimed to derive a discrete-time pricing formula for perpetual American options, assuming that the underlying asset adheres to Black-Scholes Brownian motion [15]. Subsequent research applied a similar framework to analyze demand within a manufacturing system prone to failures. The objective was to construct a financial model and devise hedging strategies for such systems. Mao and He (2009) demonstrated how the Black-Scholes model could be modified to accommodate uncertain demands in manufacturing systems with unreliable production machinery. Despite the development of theoretical approaches for pricing derivatives, pricing inconsistencies persisted. To address this, a paper introduced a practical option pricing method by integrating skewness and kurtosis adjustments into the Black-Scholes model, along with time series analysis and Artificial Neural Network (ANN) techniques[5]. The findings indicated that employing time series analysis and ANN methods enhanced the speed and accuracy of price estimation. Additionally, there was a successful implementation of the Black-Scholes model on Android smartphones and tablets, showcasing its practicality in real-time applications[16].

The Black-Scholes option pricing model derives its market value from various inputs like the underlying asset's price, exercise price, interest rate, expiration time, and stock volatility [17]. Pricing options involves a complex interplay of these factors, condensed into Greek letters, each symbolizing a distinct aspect. This complexity mirrors property pricing characteristics, where the volatility of underlying property prices remains constant, akin to a normal distribution of returns. Employing the Black-Scholes model in urban planning assumes the logarithmic normality of stock prices evolving over time as a normally distributed random variable, with each factor operating independently over time [18]. This approach permits assigning different weights to these factors, making mathematical equations a more fitting, comprehensive, and accurate method compared to techniques like ordinary least squares regression commonly used in urban planning.

The prior sections of the literature review affirm the feasibility of the Black-Scholes model in handling pricing functions across disciplines. Utilizing the Black-Scholes Model to depict the housing price trend in a metropolitan area aligns perfectly with the objectives of this study. Hence, assuming the real estate market involves at least one risky asset and one riskless asset, we consider:

(1) The risk-free interest rate, representing the return rate on the riskless asset.

(2) The random walk, indicating the instantaneous log returns of the asset price, akin to an infinitesimal random walk with a drift. This walk follows a geometric Brownian motion, maintaining constant drift and volatility. If these aspects vary over time, modifications to the Black-Scholes formula can be deduced, provided the volatility remains non-random.

(3) The asset that doesn't yield dividends.

In addition, the ideal market conditions for the proposed function adhere to the following assumptions:

(1) Absence of arbitrage opportunities.

(2) Ability to borrow and lend any cash amount, including fractional sums, at the riskless rate.

(3) Ability to buy and sell any quantity, even fractional, of the asset, including engaging in short selling.

(4) Transactions mentioned above carry no fees or costs within this frictionless market setting.

Despite the presence of a derivative security within this market, it's possible to define the security's payoff at a predetermined future date, contingent upon the value(s) assumed by the asset leading up to that specific date. Notably, the price of the derivative is entirely established at the present time. When considering the behavior of real estate prices as governed by geometric Brownian motion, characterized as the Wiener Process, the Wiener process (x_t) represents continuous random variables within a continuous time frame, yielding the following equation:

$$dx_t = x_{t+dt} - x_t = r(dt)^{1/2} \tag{3}$$

where d_t stands for Wiener process through changes of the amount of time during t, r is subject to the standard normal distribution of random variables. Equation (3) meets the Markov Property because market price quickly reflects all relevant information in the past. Since geometric Brownian and Stochastic differential equation is expressed

$$dS_t = S_{t+dt} - S_t = \alpha S_t dt + \beta S_t dx_t \tag{4}$$

which fits Equation (3) Wiener process. we can replace Equation (3) by Equation (4) where α and β are the parameters representing the rate of change of a random process average speed, α represents internal features (e.g.: house age, height, floor number) and fluctuations, β represents external environment characteristics (e.g.: number of hospitals, distance from train station, number of stores), x_t stands for Wiener process. Point S_t follows lognormal distribution and the natural logarithm $ln\ S_t$ also follows normal distribution, leaving that S_t only has a positive value. Also, the S_t and S_0 relationship can be also expressed by taking integration of Equation (4):

$$S_t = S_0 e^{\left(\left(\alpha - \frac{\beta^2}{2}\right)t + \beta X_t\right)} \tag{5}$$

Having Black-Scholes equation derived using stochastic differential equation of Ito's Lemma theorem which is also a random process, y_t presents the Ito process, leading to the random differential equation expressed as follows:

$$dy_t = y_{t+dt} - y_t = \alpha(y_t,t)dt + \beta(y_t,t)dx_t \tag{6}$$

where α and β are random parameters in the process itself. The t functions of $\alpha(y_t,t)dt$ and $\beta(y_t,t)$ are drift term and volatility terms. They still have Markov properties. Assuming that the price follows the geometric Brownian motion, the commodity prices is S_t and differentiable function $F(S_t,t)$ is:

$$dF(S_t\,,t) = \left[\frac{\partial F(S_t\,,t)}{\partial S_t}\alpha S_t + \frac{\partial F(S_t\,,t)}{\partial S_t} + \frac{1}{2}\frac{\partial^2 F(S_t\,,t)}{\partial S_t^2}(\beta S_t)^2\right]dt + \frac{\partial F(S_t\,,t)}{\partial S_t}\beta S_t dX_t \tag{7}$$

Assuming that the fixed continuous compound interest rate i in the market, V represents the value of a bank account at the time t and $V = 0$ as the starting point, we have $V_t = V_0 \times e^{it}$. Taking differential on t, thus,

$$dv_t = i\ V_0\ e^{it}dt = iV_t dt. \tag{8}$$

It is a differential equation with random bank account value where its volatility is zero in terms of Ito's process. Zero volatility implies that there is no Wiener random factor and the behavior is growth with a fixed growth rate, which corresponds to the price behavior of real estate assets.

Assuming that real estate commodities as a portfolio, we can sell a unit of $\frac{\partial F(S_t\,,t)}{\partial S_t}$related to a real estate portfolio. Then this portfolio value, P_t, is $F(S_t\,,t) - \frac{\partial F(S_t\,,t)}{\partial S_t}$. Taking first derivative, we have:

$$dP_t = dF(S_t\,,t) - \frac{\partial F(S_t\,,t)}{\partial S_t}dS_t \tag{9}$$

Using Equation (7) to rewrite Equation (9), we obtain:

$$dP_t = \left[\frac{\partial F(S_t\,,t)}{\partial t} + \frac{1}{2}\frac{\partial^2 F(S_t\,,t)}{\partial S_t^2}(\beta S_t)^2\right]dt \tag{10}$$

Equations (10) and (8) do not have the random factor dx_t., and thus, the portfolio and bank account belong to risk-free assets. Also, if we let $dP_t = iP_t d_t$ according to the concept introduced by Equation (8), we can rewrite Equation (9) to:

$$dP_t = i\left[F(S_t\,,t) - \frac{\partial F(S_t\,,t)}{\partial S_t}S_t\right]dt \tag{11}$$

Equations (10) and (11) both present procurement of a single unit of a commodity and sell $\frac{\partial F(S_t\,,t)}{\partial S_t}$ unit of the underlying real estate portfolio value. Equations (10) and (11) are equal:

$$\left[\frac{\partial F(S_t\,,t)}{\partial t} + \frac{1}{2}\frac{\partial^2 F(S_t\,,t)}{\partial S_t^2}(\beta S_t)^2\right] = i\left[F(S_t\,,t) - \frac{\partial F(S_t\,,t)}{\partial S_t}S_t\right] \tag{12}$$

And,

$$iF(S_t\,,t) = \frac{\partial F(S_t\,,t)}{\partial t} + iS_t\frac{\partial F(S_t\,,t)}{\partial S_t} + \frac{1}{2}\frac{\partial^2 F(S_t\,,t)}{\partial S_t^2}(\beta S_t)^2 \tag{13}$$

For all real estate commodities, if the price formula S_t and time t have differentiable functions $F(S_t,t)$, the differential Equation (13) must be satisfied. This equation is primarily applied to a regional or large-scale real estate pricing attractive to government, bankers and developers, who are interested in massive development or investment.

4. PRICING MODEL IMPLEMENTATION IN URBAN AND RURAL AREA

To showcase the applicability of the proposed model in addressing real-world scenarios, we conducted two comprehensive case studies. Employing convenience sampling and data sampling criteria, Taipei City in Taiwan was chosen to represent an urban area, while Sidoarjo in Indonesia was selected to represent a rural area.

4.1Model Implementation in Urban Area

With a 95% confidence level and a 10% margin of error in a 50-50 category proportion, we conducted a data sampling process that randomly selected 4,178 house transactions in the Taipei City region between 2008 and 2010 for our study. The age of houses within these transactions ranged from one to 20 years, a range chosen to mitigate significant overhauls that could substantially impact selling prices. The collected data exhibit two primary characteristics: residential aspects (internal factors) and environmental elements (external factors). The former encompasses 10 factors, including floor space, room ratios, bathrooms, land area, housing age, halls, building height, floors, and marketing duration. The second set involves 28 factors, encompassing details like proximity to Mass Rapid Transit (MRT) stations, the number of nearby MRT stations, distances to train and interchange stations, highways, entrances to highways, universities, libraries, arts centers, shopping malls, department stores, supermarkets, night markets, hospitals, police and fire stations, and parks. These factors, denoted by α and β in Equations (2) and (3) respectively, constitute external and internal considerations. Table 1 illustrates the general internal conditions affecting house procurement in Taipei's urban area, Taiwan. After removing outlier data using a threshold set at a p-value of 0.05, the maximum and minimum floor areas were 426.08 and 29.60 square meters respectively. Meanwhile, the average floor area was 109.52 square meters, with a variance of 0.3950. This suggests that housing prices in this area tend to be relatively high compared to the average salary. This is further evident in the average values for house age, room quantity, hall quantity, and bathroom quantity. The average sale period of 38.51 days indicates a relatively rapid turnover rate for house sales in Taipei's urban area, significantly influencing the α factor.

Table 1 α values by internal factors

Factors	Max.	Min.	Average	Variance	Weight
Floor area (m^2)	426.08	29.60	109.52	0.3950	0.020
Land area (m^2)	144.76	0	26.02	0.5421	0.008
House age (year)	49.30	0	22.37	0.4785	-0.055
Room (quantity)	10	1	2.76	0.3176	-0.019
Hall (quantity)	8	1	1.87	0.2191	0.027
Bathroom (quantity)	8	1	1.59	0.3922	0.094
Floor (quantity)	21	1	4.60	0.6461	0.027
Building height (m)	30	1	7.58	0.5302	0.011
Sale period (day)	602	1	38.51	1.2768	0.217
Total weight for external factors (α)			0.3448		

Table 2 predominantly presents the quantities and distances associated with external factors. Within each dataset, there are 16 external factors typically found in urban areas: MRT stations, train stations, airports, highway interchanges, expressways, universities, libraries, convention centers, outlets, department stores, supermarkets, night markets, hospitals, police stations, fire stations, and parks. A maximum walking distance of 800 meters, equating to a 15-minute walk, is designated for accessing these facilities. Among these factors, only the proximity of fire stations tends to deter house buyers, regardless of their distance. Urban buyers generally appreciate facilities like MRT stations, universities, libraries, outlets, department stores, and night markets, yet they prefer to maintain a certain distance from them, prioritizing a quiet and secure living environment.

The calculation of α and β considers the weight and variance outlined in Tables 1 and 2, where variance serves as a random variable describing the extent of dispersion.

Table 2 β value by external factors

Factors	**Average**	**Variance**	**Weight**
Distance to MRT station	286.094	0.90	-0.053
Quantity of MRT station nearby	1.181	0.83	0.106
Distance to train station	58.896	3.04	0.141
Distance to airport	27.564	4.38	0.047
Distance to highway interchange	52.350	3.25	-0.014
Distance to expressway	218.285	1.16	0.189
Quantity of expressway interchange nearby	0.807	1.00	-0.201
Distance to university	262.138	1.08	-0.015
Quantity of university nearby	0.957	1.17	0.282
Distance to library	357.036	0.64	-0.106
Quantity of library nearby	1.724	0.70	0.044
Distance to conventional center	245.720	1.06	0.048
Quantity of conventional center nearby	1.404	1.26	0.009
Distance to outlet	116.622	1.96	-0.006
Quantity of outlet nearby	0.266	1.87	0.032
Distance to department store	151.884	1.60	-0.003
Quantity of department store nearby	0.924	1.93	0.102
Distance to supermarket	246.853	0.61	0.026
Quantity of supermarket nearby	5.598	0.48	0.215
Distance to night market	118.329	1.86	-0.073
Quantity of night market nearby	0.313	1.67	0.190
Distance to hospital	295.741	0.88	0.048
Quantity of hospitals nearby	1.365	4.55	0.087
Distance to police station	356.668	0.60	0.096
Quantity of police station nearby	2.262	0.84	-0.249
Distance to fire station	303.433	0.90	-0.071
Quantity of fire station nearby	0.826	0.92	-0.034
Distance to park	169.350	1.42	-0.115

Factors	Average	Variance	Weight
Total weight for external factors (β)			1.5552

In essence, it represents the average distance between all measured data points and their average value. A higher value indicates a greater degree of dispersion, signifying a larger gap between reality and people's expectations. By substituting the values of α and β into Equation (3) and setting time (t) from 0 to 3, reflecting house prices between 2007 and 2010, Table 3 displays the ranges for the Wiener process (xt) and housing prices (St) in Taipei City. At t=0.5 (06/2007), for instance, housing prices in Taipei City ranged from $219,313 to $44,407 New Taiwan Dollars (NTD) per square meter, excluding those falling within the upper and lower 5% of house prices. Despite an average 18.57% increase in housing prices over three years, the range narrowed, with the lower boundary increasing more than the upper limit. Reasonable price growth in regional urban areas typically ranges from 2.1% to 21.5%, contingent upon district locations. This confirms the gradual unaffordability of housing in urban areas of Taiwan.

Table 3 Housing price range in Taipei area

Time (t)	S_t min ($)	S_t max ($)	S_t average ($)	x_t
0.5 (07/2007)	44,407	219,313	98,887	-0.215439 ~ 0.811497
1.0 (01/2008)	46,888	281,597	100,975	0.097448 ~ 1.250171
1.5 (07/2008)	49,792	286,407	104,635	0.414025 ~ 1.538999
2.0 (01/2009)	39,476	119,548	82,401	0.542693 ~ 1.255149
2.5 (07/2009)	68,335	159,387	100,128	1.153437 ~ 1.718026
3.0 (01/2010)	69,182	278,875	117,249	1.459316 ~ 2.355674

Figure 1 showcases the trend in house prices within Taipei's urban area, while Figure 2 displays the value range for xt. Evidently, the Wiener process indicates an upward trend in urban housing prices. These findings not only align with real-world observations but also validate the viability of the proposed model, which integrates macroeconomics and environmental features to explain housing price behaviors.

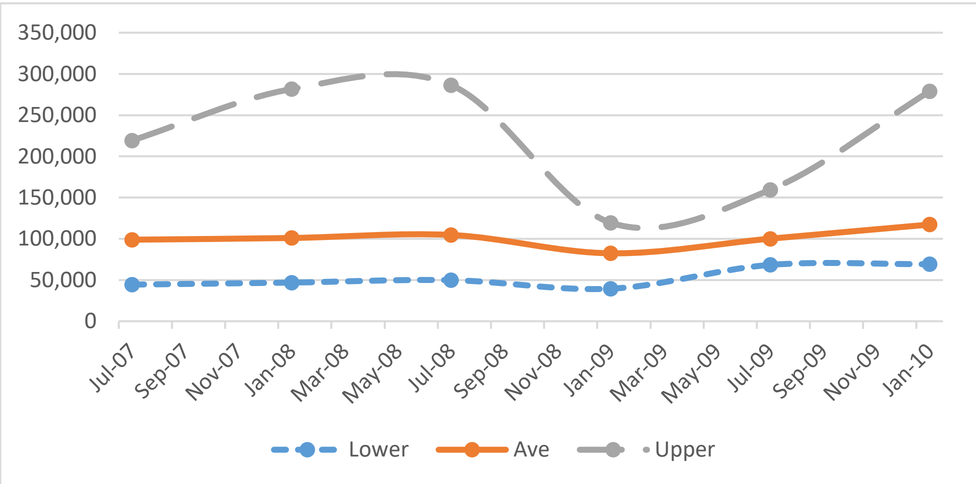

Figure 1 Land price range in Taipei area during 2008 to 2010

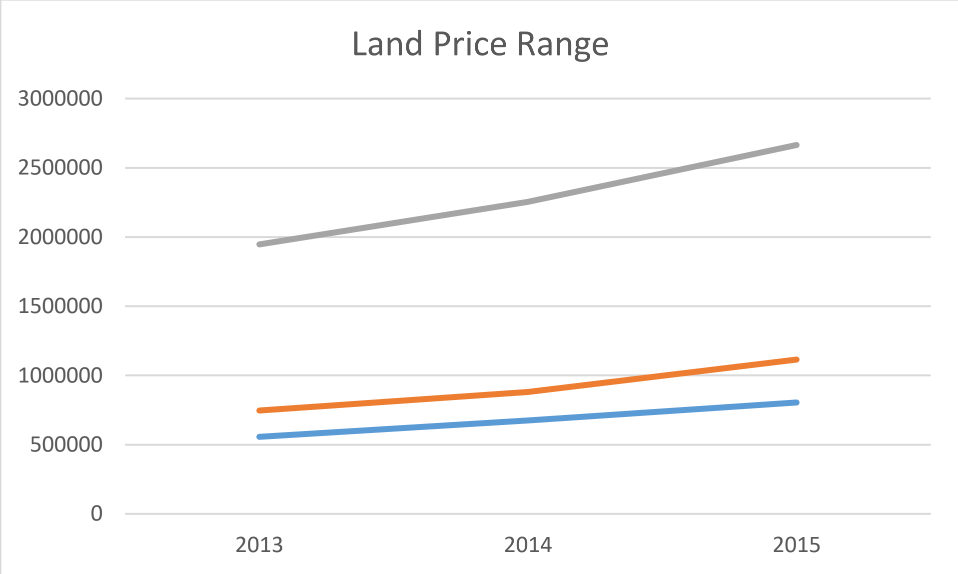

Figure 2 Land price range in Sidoarjo area during 2013-2015

4.2 Model implementation in rural area

Utilizing convenience sampling, the rural case study selected a satellite city, Sidoarjo in Indonesia. The population count, totaling 9,393 land plots, was determined through digitizing features on ArcGIS 10.1. The majority of this population shares a predominant characteristic: a landscape dominated by residential buildings and rice fields. Employing Slovin's formula at a 95% confidence level, the required sample size for this research was calculated at 384 samples. These land plots were selected randomly for analysis. In this study, land prices are assumed to remain consistent without sudden changes or market surprises. Table 4 outlines the corresponding descriptions, values, and weights essential for consideration. The term 'nearby' in the tables indicates a maximum distance of 800 meters, approximately a 15-minute walking distance. However, some facilities listed exhibit an average distance exceeding 1000 meters. This is attributed to the residents' preference for using their own vehicles to reach these amenities within the study area. Notably, there are negative weights attributed to physical attributes. This signifies that the farther a property is located from flood-prone or mudflow areas, the higher its perceived value should be. These factors were computed based on weight and variance, where variance serves as a random variable depicting the extent of dispersion. A higher variance value signifies a greater discrepancy between reality and people's expectations.

Table 4 Factors with its values and weights

Factors	Average	Variance	Weight
Distance to arterial road	456.3186	1.19	0.272691
Distance to train station	460.4532	1.91	0.271888
Distance to library	1095.276	2.28	0.26747
Distance to downtown	1095.276	2.28	0.284739
Distance to outlet	1095.276	2.28	0.269076
Quantity of department store nearby	0.0528	0.48	0.276707
Distance to supermarket	1258.484	6.43	0.27751
Distance to healthcare facility	509.696	1.29	0.269076
Quantity of healthcare facility nearby	0.7813	0.48	0.264257
Distance to park	306.5509	7.85	0.270683

Factors	Average	Variance	Weight
Distance to flood-prone areas	1784.924	5.74	-0.272289
Distance to mud flow areas	1789.58	8.56	-0.273092
Distance to planned industrial park	372.6411	1.33	0.273092
Distance to planned road network	1000.388	2.91	0.281124

By substituting the factor values into Equation and setting time (t) between 2013 and 2015 to represent the land price during this period, Table 5 displays the ranges for the Wiener process (xt) and land prices (St) in the study area. For instance, in 2013, the land price in the study area ranged from IDR 190,000 to 1,200,000 per square meter, excluding those in the upper and lower 5% of land prices. Over three years, the average land price increased by 21.47%. When applied to a specific plot of land within the study area, the model predicted a land price of 1,388,719 IDR per square meter for the year 2015, while the actual price stood at 1,300,000 IDR per square meter, resulting in an accuracy rate of 93.6%.

Table 5 Minimum, average, and maximum land price during 2013-2015

Time (t)	S_t min ($)	S_t max ($)	S_t average ($)	x_t
2013	190,000	1,200,000	556661.5	0.14459 ~ 0.91323
2014	205,000	1,375,000	674911.5	0.15601 ~ 1.04641
2015	310,000	1,550,000	804630.2	0.23591 ~ 1.17959

5. DISCUSSIONS

Initially built upon geometric Brownian motion to capture real estate price behaviors, the property price model equations adhere to the Markov Property, swiftly incorporating all pertinent past information into current market prices. This model incorporates parameters α and β, signifying the speed of change in an average random process. α delineates internal features and fluctuations, while β characterizes external environmental traits. The derivation of the Black-Scholes equation employs the stochastic differential equation via Ito's Lemma theorem. This equation constitutes a differential equation involving a random bank account value, where its volatility registers as zero in terms of Ito's process. Zero volatility denotes the absence of a Wiener random factor, indicating a growth pattern with a fixed growth rate, aligning with the behavior of property assets' prices. The case study underscores disparities in the application of the property price model, noting differences between its implementation for urban housing and rural land assessments.

While the equations remain constant, each case possesses its unique characteristics necessitating distinct adjustments. The primary disparity arises from the diverse nature of real properties, as not all properties exhibit both α and β parameters. For instance, land properties lack internal characteristics, rendering α parameters irrelevant in these cases. This absence of parameters impacts the proposed model, resulting in slight yet discernible differences in accuracy between the two cases. On average, property prices increased by 18.57% in urban areas with an accuracy rate of 95%, while in rural areas, the average increase was 21.47% with an accuracy rate of 93.6%. Both areas exhibit an upward trend in property prices. However, the urban case experiences fluctuations in the upper price range, while the rural case demonstrates a more linear progression. Dissimilarities in β parameters affecting property prices stem from environmental variations between urban and rural settings. Certain factors cannot be applied due to distinct characteristics between these areas. For instance, environmental factors like MRT stations, convention centers, and night markets are unavailable as parameters in rural areas. MRT stations, considered high-end facilities, are mostly absent in rural regions, including Indonesia, where the metro system was still under development in 2018. However, the accessibility facilitated by arterial roads remains a crucial factor, enhancing the economic value of a given region. Additionally, distinct environmental parameters, such as mudflow, significantly influence land prices. The topography and physical attributes of each area directly impact land prices, where proximity to mudflow areas often correlates with lower land prices.

6. CONCLUSION

The study formulated a land pricing model based on the Black and Scholes theory, integrating the concept of environmental features, and subsequently compared its application in both urban and rural settings. A pivotal contribution of this study lies in presenting a mathematical model that combines macroeconomics and environmental factors to determine regional property prices. Building upon the Black and Scholes theory, evidence indicates correlations between theory parameters and environmental features, facilitating the development of a model theoretically sufficient to estimate land prices within specific areas.

The model implementation targeted two distinct locations. In the urban case, the model analyzed real estate data from the Taipei City region, where a total of 4,178 real estate transactions between 2008 and 2010 were randomly sampled, meeting the criteria of a 95% confidence level with a 10% error limit in a 50-50 category proportion. Meanwhile, in the rural area, the model assessed land price data adjacent to Porong Arterial Road in Sidoarjo, Indonesia, covering the period from 2013 to 2015, utilizing a convenience sample of 9,393 land plots based on the same 95% confidence level criterion. The model introduces a pricing function predicting house prices along with upper and lower boundaries, offering insights into better and worse scenarios. It also delineates tendencies elucidating the developmental potential of a given region. When applied to both housing and land properties, the model demonstrates an accuracy range between 93.6% and 95%. The significant contribution lies in demonstrating the model's applicability in diverse settings, spanning urban and rural areas across both developed and developing countries. Its adaptability, contingent upon adjustments specific to property characteristics and regional nuances, renders it highly versatile. Replicating the proposed approach in other regions merely involves following the steps outlined in the implementation section.

The discoveries from this study hold significance for both academia and industry, aiding in the establishment of pricing models specific to regions and in determining profit margins for real estate developers. The study introduces a mathematical model merging the Black and Scholes theory with the hedonic pricing method to elucidate real estate behavior in urban and rural settings. It's worth noting that the ranges for the Wiener process might undergo potential adjustments. Additionally, various factors may influence the land price model, necessitating further empirical evidence for exploration. Understanding how the property market operates and evolves allows for a deeper comprehension of the relationship between external factors, land price, and land value. This comprehension equips urban planners to optimize land utilization, strike a balance in regional land prices, and ultimately enhance land efficiency within a given region.

ACKNOWLEDGMENTS

The authors extend their gratitude for the partially support provided for this research by the Taiwan Ministry of Science and Technology (MOST) / National Science and Technology Council (NSTC) under the grant numbers MOST-108-2221-E-008 -002 -MY3, MOST-109-2622-E-008 -018 -CC2, MOST- 110-2622-E-008 -018 -CC2, MOST-110-2221-E-008 -052 -MY3, NSTC-111-2622-E-008-017, and NSTC-111-2221-E-008 -027 -MY3. It is important to note that any opinions, findings, conclusions, and recommendations presented in this paper solely belong to the authors and do not necessarily reflect the perspectives of the MOST/NSTC.

REFERENCES

[1] Levkovich, O., J. Rouwendal, and R. Van Marwijk (2016). "The effects of highway development on housing prices." *Transportation*, 43(2), 379-405.

[2] Maddison, D. (2000). "A hedonic analysis of agricultural land prices in England and Wales." *European Review of Agricultural Economics*, 27(4), 519-532.

[3] Economics, G.L.A. (2016). "Economic Evidence Base for London 2016." Greater London Authority, London.

[4] Jiang, L., P.C. Phillips, and J. Yu (2014). "A new hedonic regression for real estate prices applied to the Singapore

residential market."

[5] Liu, K. and X. Wang "A pragmatical option pricing method combining Black-Scholes formula, time series analysis and artificial neural network." *Proc., 2013 Ninth International Conference on Computational Intelligence and Security*, IEEE, 149-153.

[6] Albeverio, S., A. Popovici, and V. Steblovskaya (2006). "A numerical analysis of the extended Black–Scholes model." *International Journal of Theoretical and Applied Finance*, 9(01), 69-89.

[7] Kong, F., H. Yin, and N. Nakagoshi (2007). "Using GIS and landscape metrics in the hedonic price modeling of the amenity value of urban green space: A case study in Jinan City, China." *Landscape and urban planning*, 79(3-4), 240-252.

[8] Guo, B. and K. Sun "Research on real estate price based on hedonic analyze among cities." *Proc., 2010 International Conference on System Science, Engineering Design and Manufacturing Informatization*, IEEE, 218-221.

[9] Shonkwiler, J.S. and J.E. Reynolds (1986). "A note on the use of hedonic price models in the analysis of land prices at the urban fringe." *Land economics*, 62(1), 58-63.

[10] Chung, S.L. and M. Shackleton (2002). "The binomial Black–Scholes model and the Greeks." *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, 22(2), 143-153.

[11] Pelsser, A. and T.C. Vorst (1994). "The binomial model and the Greeks." *Journal of Derivatives*, 1(3), 45-49.

[12] FA, N. (1996). "FX option theta and the joker'effect." *Derivatives Quarterly*, 56-58.

[13] Emery, D.R., W. Guo, and T. Su (2008). "A closer look at Black–Scholes option thetas." *Journal of economics and finance*, 32(1), 59-74.

[14] Ghaffari, R. and B. Venkatesh "Wind energy forecast error estimation using black & scholes mathematical model." *Proc., 2014 IEEE 27th Canadian Conference on Electrical and Computer Engineering (CCECE)*, IEEE, 1-6.

[15] Huang, W., S. Li, and S. Zhang "Pricing Perpetual American Option under the Fractional Black-Scholes Model." *Proc., 2010 Third International Conference on Business Intelligence and Financial Engineering*, IEEE, 165-169.

[16] Koh, H.L. and S.Y. Teh "Learning Black Scholes option pricing the fun way via mobile apps." *Proc., Proceedings of 2013 IEEE International Conference on Teaching, Assessment and Learning for Engineering (TALE)*, IEEE, 192-195.

[17] Edeki, S.O., O.O. Ugbebor, and E.A. Owoloko (2015). "Analytical solutions of the Black–Scholes pricing model for European option valuation via a projected differential transformation method." *Entropy*, 17(11), 7510-7521.

[18] Hedman, K.W. and G.B. Sheblé "Comparing hedging methods for wind power: Using pumped storage hydro units vs. options purchasing." *Proc., 2006 International Conference on Probabilistic Methods Applied to Power Systems*, IEEE, 1-6.